Centrality and  $E_T$  Fluctuations from p+Be to Au+Au at AGS Energies M. J. TANNENBAUM for The E802 Collaboration Brookhaven National Laboratory Upton, NY 11973-5000 USA

#### **ABSTRACT**

Measurements by the E802 Collaboration of the A-dependence and pseudorapidity interval  $(\delta \eta)$  dependence of mid-rapidity  $E_T$  distributions in a halfazimuth electromagnetic calorimeter are presented for p+Be, p+Au, O+Cu, Si+Au and Au+Au collisions at the BNL-AGS. The shapes of  $E_T$  distributions as characterized by the p parameter of a Gamma Distribution are observed to systematically vary with the size of the  $\delta\eta$  interval. However, in contrast to the situation for Multiplicity Distributions where the shape as characterized by the NBD parameter  $1/k(\delta \eta)$  can be related to the 2-particle short-range correlation length, there is at present no theoretical framework to relate the systematic variation in the shapes of  $E_T$  distributions to other physical quantities. A new way of plotting  $E_T$  distributions was found. If the  $E_T(\delta \eta)$  distributions in any aperture are plotted in units of the measured  $\langle E_T(\delta \eta) \rangle$  in the same aperture for p+Au collisions, then the plots as a function of  $\delta\eta$  become nearly universal in the physically meaningful units of 'number of average p+Au collisions', effectively Wounded Projectile Nucleons. This shows that the centrality characterization remains valid even in relatively small mid-rapidity pseudo-rapidity intervals. The relative effects of acceptance (due to the limited aperture), nuclear geometry and the underlying fundamental fluctuations on the detailed shapes of the upper edges of the  $E_T$  spectra are examined.

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#### E802/E859/E866 Midrapidity $E_T$ Measurements In PbGlass Electromagnetic Calorimeter

$$\heartsuit E_T \equiv \sum_{\text{photons}} E_{\gamma} \sin \theta + \sum_{\text{charged, } \beta \geq 0.8} (0.45 \text{ GeV}) \times \sin \theta$$

 $\heartsuit$  The measure is precise but not <u>accurate</u>.

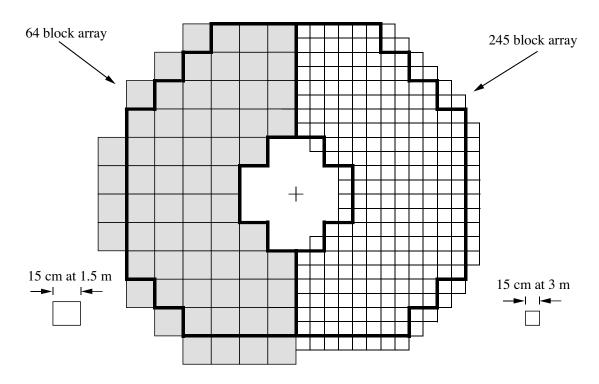


Fig. 1

Figure 1: E802/E859/E866 PbGl Detector. The  $\delta\eta$  dependence is made using the more highly segmented right side of the calorimeter.

#### $E_T$ and Multiplicity Distributions

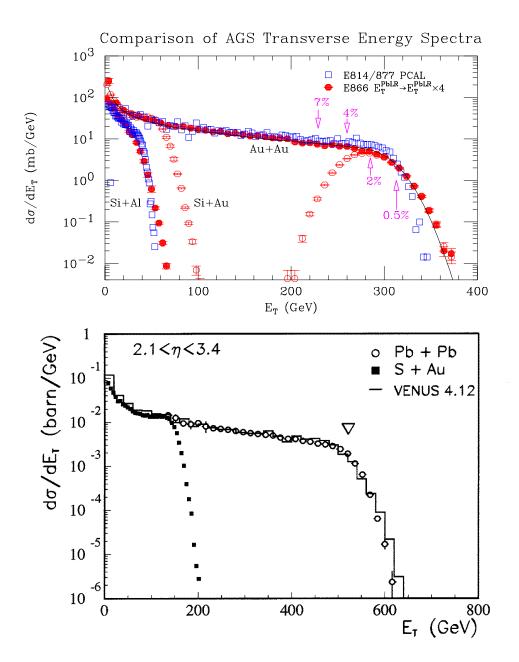


Figure 2: Top(t): AGS measurements—E814/E877  $E_T$  spectra in a full-azimuth hadron calorimeter compared to E802/E866 full azimuth  $E_T$  spectra in an EM calorimeter covering a smaller pseudorapidity interval. E802/E866 data include a central Au+Au spectrum defined by the 4 %-ile of the distribution in a Zero Degree Calorimeter (ZCAL). The solid line on the E802/E866 Au+Au data is an empirical calculation (see text). Bottom(b): CERN measurement—NA49 mid-rapidity  $E_T$  spectrum in a full-azimuth hadron calorimeter.

### 'Centrality' and percentiles of $E_T$ distributions

- $\heartsuit$  At the AGS E802/E866 and E814/E877 use  $E_T$  in an Electromagnetic or hadronic calorimeter to define 'centrality', typically by a certain upper percentile of the distribution.
- $\heartsuit$  The less constrained distributions measured by E802/E866 in an EM calorimeter covering  $1.3 \le \eta \le 2.4$  (and scaled by a factor of 4 in  $E_T$  for visual effect) fluctuate more than the more constrained distributions measured by E814/E877 in a hadron calorimeter covering  $0.83 \le \eta \le 4.7$ .
- $\heartsuit$  **But** for the most part the distributions are very similar in shape, and therefore in Centrality Definition.
- $\heartsuit$  The  $E_T$  spectra measured by NA49 in a mid-rapidity hadron calorimeter at CERN show the more rounded upper edges, similar to the AGS mid-rapidity distributions.
- $\heartsuit$  The  $E_T$  emission in Au+Au (Pb+Pb) relative to Si+Au can be simply read from Fig. 2 and is clearly  $\sim 20\%$  larger at AGS energies compared to CERN. This is likely a reflection of the 'stopping' at AGS energies which depresses the energy emission from successive collisions in asymmetric (Si+Au) reactions compared to the symmetric case (Si+Al).

### INTERMITTENCY isn't CHAOS E802 O+Cu Central Multiplicity Distributions vs. $\delta\eta$

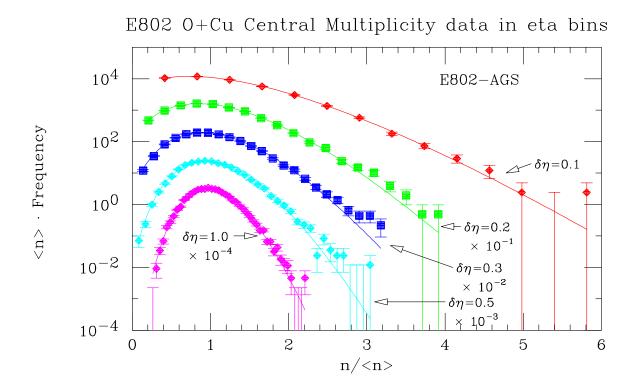


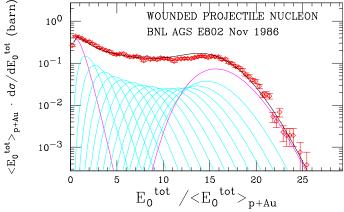
Figure 3: Multiplicity distributions measured in  $^{16}\text{O}+\text{Cu}$  central collisions as a function of the interval  $\delta\eta$  (indicated), scaled by < n > on the interval, for the case when all 16 incident nucleons have interacted as determined by the ZCAL.

- $\heartsuit$  The shape of the charged multiplicity distribution varies from nearly exponential for  $\delta \eta = 0.1$  to nearly gaussian for  $\delta \eta = 1.0$ .
- $\heartsuit$  The shape of multiplicity distributions, as parameterized by Normalized Factorial Cumulants  $K_q(\delta\eta)$  or by the NBD parameter  $1/k(=K_2)$ , is determined by 2-particle correlations in an elegant theoretical framework.
- $\heartsuit$  One assumes that the same effect, the variation in shape as a function of the pseudorapidity interval,  $\delta\eta$  must occur with  $E_T$  distributions, but would likely be different in detail in calorimeters.

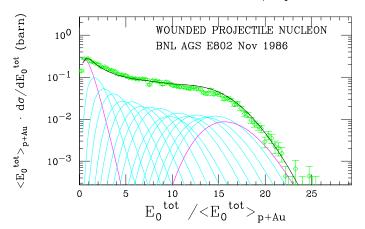
#### Mid-Rapidity $E_T$ distributions

- $\heartsuit$  As  $E_T$  measurements in limited solid angles have recently become quite popular to characterize the 'centrality' of RHI collisions, systematic investigations were made of the A dependence of such mid-rapidity  $E_T$  distributions as a function of the  $\delta\eta$  interval.
- An important issue is that models have proved useful in the understanding of the detailed shape of  $E_T$  distributions in B+A collisions as a sum of independent p+A collisions weighted according to the 'geometric' probability of the number of total or projectile participants in the reaction. If the 'shape' of  $E_T$  distributions were controlled by a correlation length and strength which changed with the number of participants differently from the effect of random combinations, then these simple models would make no sense, and, in particular, would fail to reproduce the shapes of the upper edges of the spectra. OXYGEN + Au at 14.5 GeV/c per Nucleon

WOUNDED PROJECTILE NUCLEON 100 BNL AGS E802 Nov 1986  $10^{-1}$ 



OXYGEN + Cu AT 14.5 GeV/c per Nucleon



#### Physics or Acceptance?

 $\heartsuit$  It is also conceivable that the saturation of the upper edges of the E802  $E_T$  spectra at AGS energies could be an artifact of the limited angular  $(\eta)$  acceptance.

 $\heartsuit$  In heavy ion collisions, the pseudorapdidity acceptance is an issue because naive models of successive collisions predict that the rapidity of the c.m. system, e.g. for a given projectile nucleon, shifts towards the target rapidity after each collision with a target nucleon—and vice-versa—such that the maximum in dn/dy and presumably  $dE_T/d\eta$  moves towards the larger nucleus in an asymmetric B+A reaction. Therefore, an important issue to address is whether and how the pseudorapidity acceptance of  $E_T$  distributions affects the physical interpretion of the measurement.

♡ It is known that the projectile dependence of a reaction is emphasized by measurements in the projectile fragmentation region, while the target dependence is emphasized by measurements in the target fragmentation region—thus, mid-rapidity measurements might represent a reasonable global average.

 $\heartsuit$  An additional issue is whether the shapes of mid-rapidity  $E_T$  distributions change with the interval,  $\delta\eta$ .

#### Mid-Rapidity $E_T$ distributions vs $\delta \eta$

The  $E_T$  distributions (in  $\Delta \phi = \pi$ ) were measured for successively smaller  $\delta \eta$  intervals centered on  $\eta|_0 \sim 1.86$ :  $1.25 \leq \eta \leq 2.50$  (the full  $\eta$ -acceptance of the calorimeter);  $1.38 \leq \eta \leq 2.34$ ;  $1.54 \leq \eta \leq 2.18$ ;  $1.70 \leq \eta \leq 2.02$ ;  $1.70 \leq \eta \leq 1.86$ .

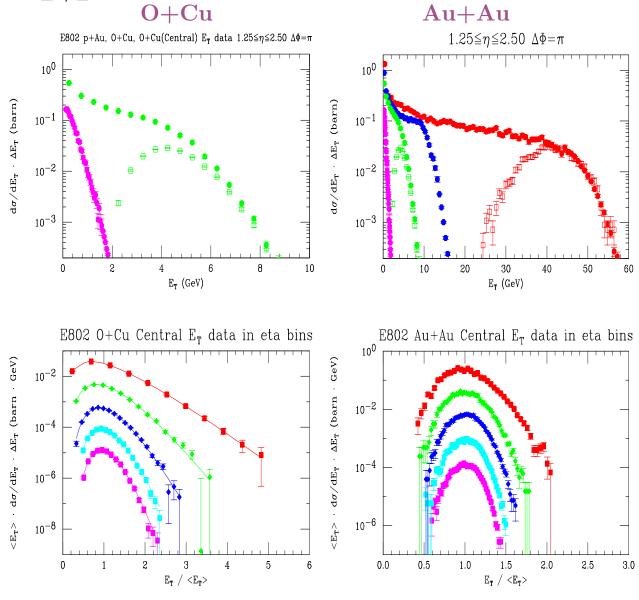


Figure 4: Top: E802 mid-rapidity  $E_T$  distributions ( $\Delta \phi = \pi$ ) for the interval 1.25  $\leq \eta \leq$  2.50. (Left) p+Au, O+Cu, O+Cu(ZCAL); (Right) previous data, plus Si+Au, Au+Au, Au+Au(ZCAL). Bottom: Central (ZCAL)  $E_T$  distributions for the five  $\delta \eta$  intervals, normalized by  $\langle E_T \rangle$  on the interval. O+Cu (left), Au+Au (right). The curves are fits to  $\Gamma$  distributions.

Evidently, the shapes of the upper edges of  $E_T$  distributions change with  $\delta\eta$ , similarly to multiplicity.

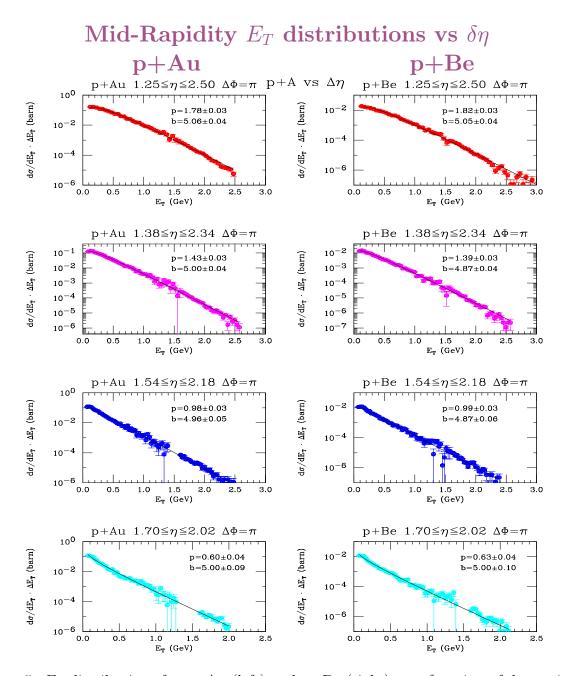


Figure 5:  $E_T$  distributions for p+Au (left) and p+Be (right) as a function of decreasing  $\delta \eta$  from top to bottom. Adjacent p+Au and p+Be plots have the same  $\delta \eta$ .

The original E802 measurements showed that the mid-rapidity  $E_T$  spectra of p+Au, p+Cu, p+Al and p+Be all exhibit the same shape over roughly 5 decades of cross section—no obvious multiple collisions effects were evident at mid-rapidity for p+A at AGS energies. In the present measurement, as the  $\delta\eta$  interval is reduced, the shapes of the  $E_T$  spectra clearly change with  $\delta\eta$  for both p+Au and p+Be; but in each  $\delta\eta$  interval, the shapes of the p+Au and p+Be distributions remain essentially **identical** with each other.

# p parameter of Gamma Fits in B+A and $\langle E_T \rangle_{\rm p+A} = p/b$ as a function of $\delta \eta$

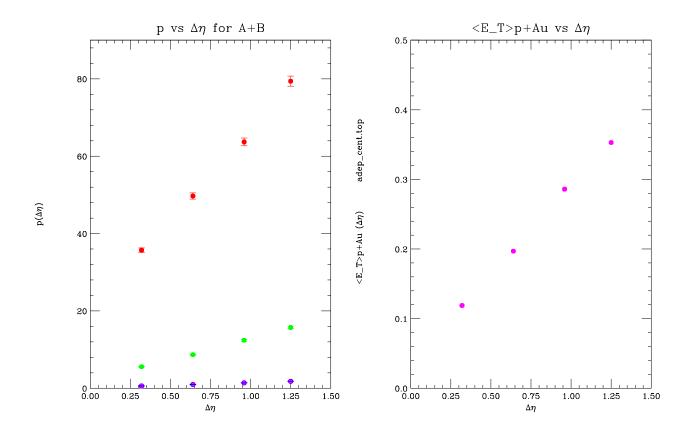


Figure 6: (left) Gamma Fit parameter p as a function of  $\delta\eta$  interval for p+Be (Blue), p+Au (Magenta), O+Cu (ZCAL) (green), Au+Au (ZCAL) (red). (right)  $\langle E_T \rangle_{p+A}$  as a function of  $\delta\eta$ .

- One Data show nice systematic behavior.
- $\heartsuit$  No theoretical framework was found in the literature to help further interpret these results.

## The Wounded Projectile Participant Model Is Preserved as a function of $\delta\eta^{-1}$

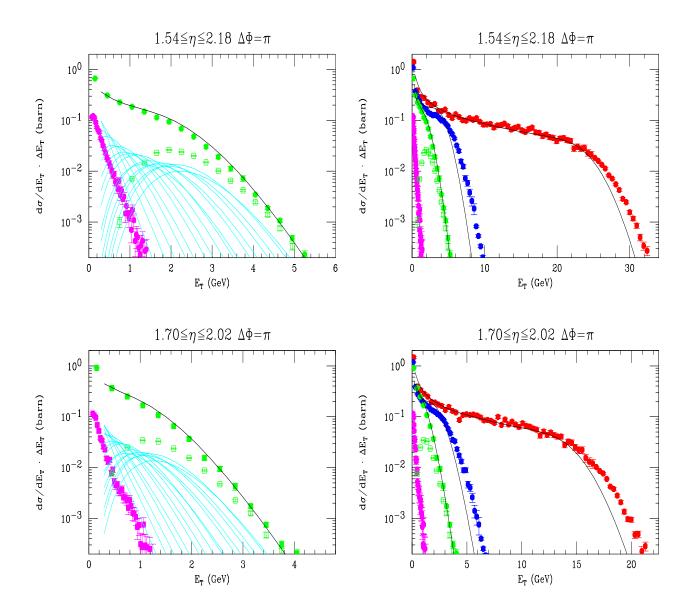


Figure 7: WPNM calculations (lines) for the two smallest  $\delta\eta$  intervals for (left) O+Cu, (right) Si+Au, Au+Au. Components are shown for O+Cu. Data shown are (left) p+Au, O+Cu, O+Cu (ZCAL); (right) same data plus Si+Au, Au+Au.

<sup>&</sup>lt;sup>1</sup>As smaller and smaller  $\delta\eta$  intervals are used for the  $E_T$  spectra, the probability  $p_0$  for a p+Au reaction to produce zero signal on the interval becomes larger and larger. This effect is easily measured from the ratio of detected cross section in the  $\delta\eta$  interval to the inelastic p+Au cross section (1662mb), and must be taken into account when performing the WPNM.

## $E_T$ Plots with WPN model for all four $\delta \eta$ intervals

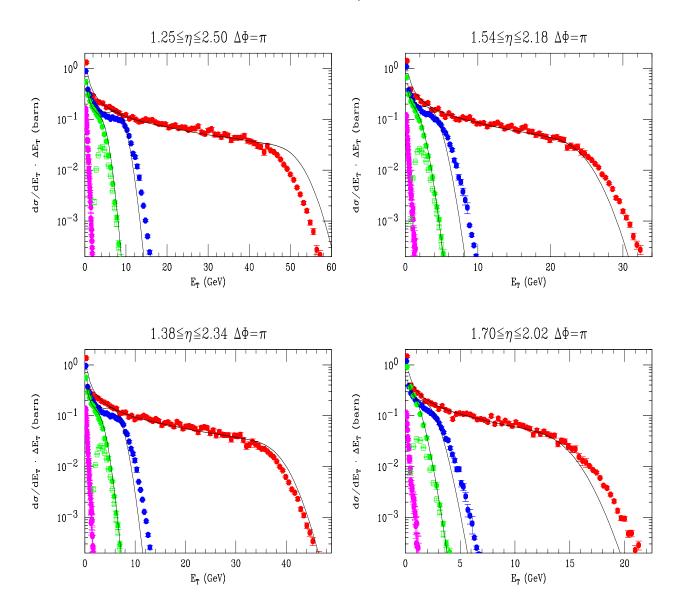


Figure 8:  $E_T$  distributions ( $\Delta \phi = \pi$ ) for the four  $\delta \eta$  intervals indicated, for p+Au, O+Cu, O+Cu (ZCAL), Si+Au, Au+Au with WPNM calculations (lines) shown. This is Fig. 7 with all intervals shown.

#### A Weak Point...

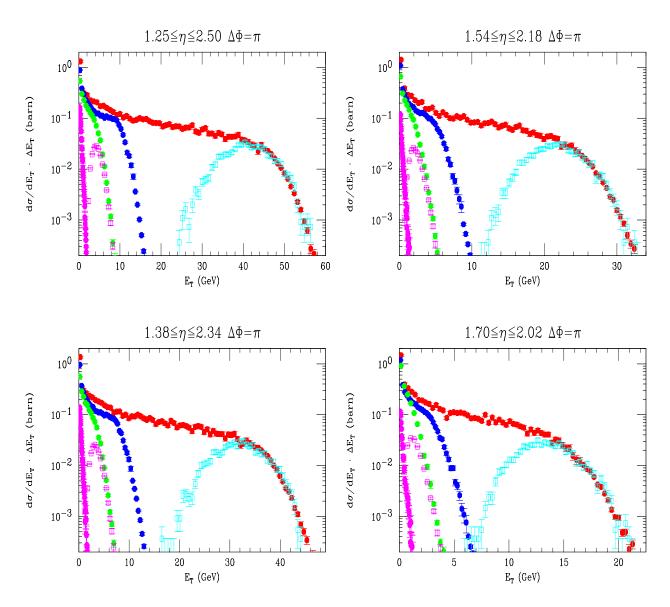


Figure 9:  $E_T$  distributions ( $\Delta \phi = \pi$ ) for the four  $\delta \eta$  intervals indicated, for p+Au, O+Cu, O+Cu (ZCAL), Si+Au, Au+Au, Au+Au (ZCAL). The plotting ranges are chosen by eye to make all the plots look similar.

 $\heartsuit$  One problem with the limited aperture, electromagnetic energy event characterization in comparison to ' $4\pi$ ' hadron calorimeters is the difficulty in relating the endpoints of the energy spectra—i.e.  $\sim 32$  GeV for  $\delta \eta = 0.64$ ,  $\sim 22$  GeV for  $\delta \eta = 0.32$ —to the total available energy for the reaction.

#### Leads to an Improved $E_T$ Plot

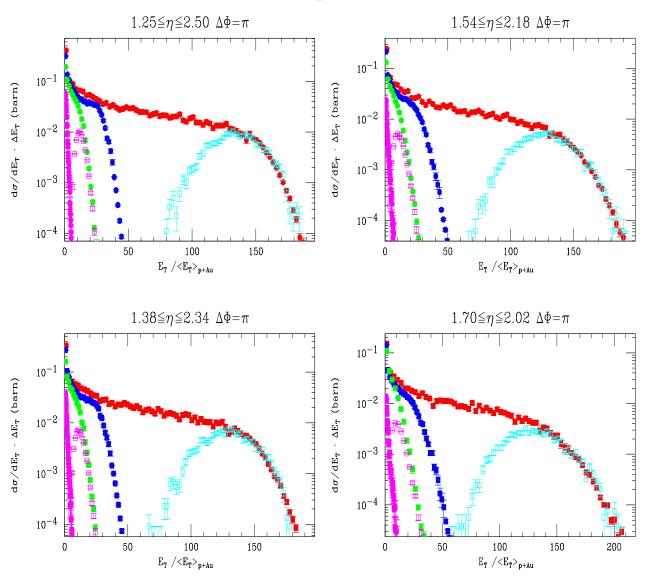


Figure 10:  $E_T$  distributions ( $\Delta \phi = \pi$ ) for the four  $\delta \eta$  intervals indicated, for p+Au, O+Cu, O+Cu (ZCAL), Si+Au, Au+Au, Au+Au (ZCAL), where the  $E_T$  scale is normalized by the measured  $\langle E_T \rangle_{\rm p+Au}$  on the interval. The Au+Au  $E_T$  has been scaled up by a factor of 1.155 to correspond to 14.6 GeV/c, see [PL **B332**, 258 (1994)].

 $\heartsuit$  When the energy scale for each aperture is plotted in units of the measured  $\langle E_T \rangle$  in the same aperture for p+Au collisions (or p-p, if available), the situation changes **dramatically**. The dynamics of the reaction, in terms of projectile (or total) participants, can now be read directly from the figure.

- $\heartsuit$  The dynamics of the reaction can now be read directly from the figure.
- $\heartsuit$  The p+Au  $E_T$  distribution goes out to  $\sim 7$  mean values over 3 orders of magnitude for  $\delta \eta = 0.64$ , and to  $\sim 10$  mean values over 3 orders of magnitude for  $\delta \eta = 0.32$ , i.e. the fluctuations around the mean increase with decreasing aperture.
- $\heartsuit$  The knees of the O+Cu and Si+Au  $E_T$  distributions occur roughly at 16 and 28 times  $\langle E_T \rangle_{\rm p+Au}$  for both apertures, corresponding to the A of the projectiles, but the knee of the Au+Au distribution is roughly 150 and clearly not  $A_{\rm Au} = 197$ , apparently indicating some shadowing.
- $\heartsuit$  Detailed analysis shows that this is actually an acceptance effect in the limited aperture ( $\epsilon^{197}$  tends to be considerably less than unity for most reasonable values of  $\epsilon$ ), so that the 'Wounded Projectile Nucleon' actually represents the measurements rather well in all the intervals studied around mid-rapidity at the AGS.

#### Other Improved $E_T$ Plots

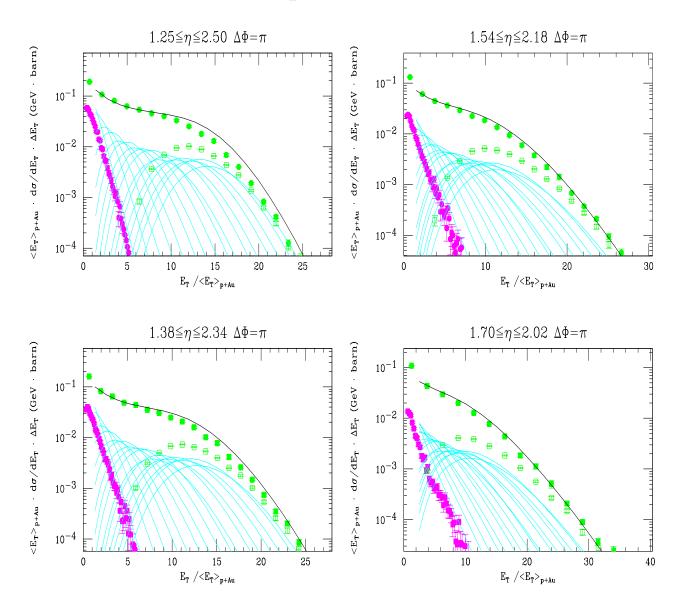


Figure 11:  $E_T$  distributions ( $\Delta \phi = \pi$ ) for the four  $\delta \eta$  intervals indicated, for p+Au, O+Cu, O+Cu (ZCAL), Si+Au, where the  $E_T$  scale is normalized by the measured  $\langle E_T \rangle_{\rm p+Au}$  on the interval.

 $\heartsuit$  Also note the 'famous' plot shown on page 6 is plotted in the improved format.

#### $E_T$ for { p,O,Si,Au} + Au, O+Cu Details of WPNM for Au+Au Upper Edge

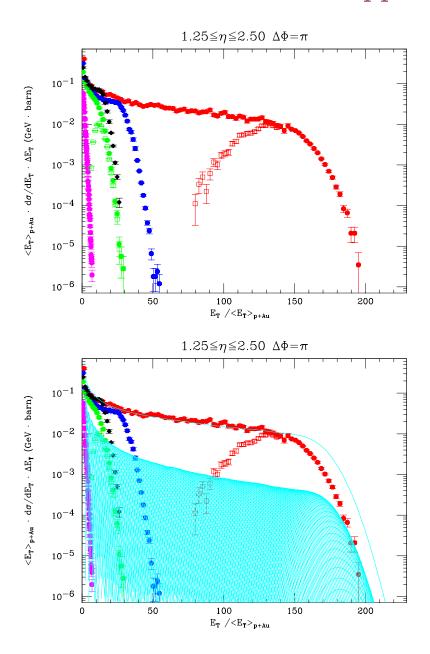


Figure 12: Top(t):  $E_T$  distributions ( $\Delta \phi = \pi$ ) in 1.25  $\leq \delta \eta \leq$  2.50 for p+Au, O+Cu, O+Cu (ZCAL), O+Au, Si+Au, Au+Au, Au+Au (ZCAL). Bottom (b) the same with WPN model for Au+Au with all components shown

## Variation of upper edge of WPNM with $\epsilon$ and p+Au shape (with fixed $\langle E_T \rangle|_{p+Au}$ )

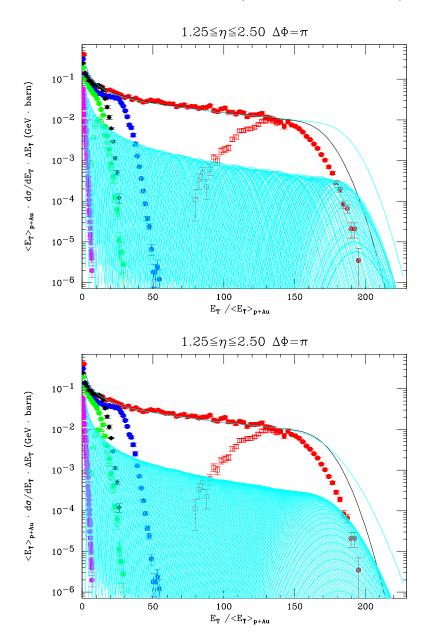


Figure 13: Top(t):  $E_T$  distributions ( $\Delta \phi = \pi$ ) in 1.25  $\leq \delta \eta \leq$  2.50 and Au+Au WPN calculation from Fig. 12 (black line) for 2 variations: (top)  $\epsilon \to 1.00$ , (bottom) underlying p+Au  $\Gamma(p,b)$ : vary p,b keeping  $\langle E_T \rangle|_{p+Au}$  fixed  $(p \to p/2, b \to b/2)$ .

The shape of upper edge is not sensitive to  $\epsilon$  but the position is. For fixed  $\langle E_T \rangle|_{p+Au} = p/b$ , the upper edge flattens as b flattens (decreases) but the 'knee' remains unchanged.

#### $A_p$ dependence (4%-ile) at Mid-Rapidity for 14.6 A GeV/c

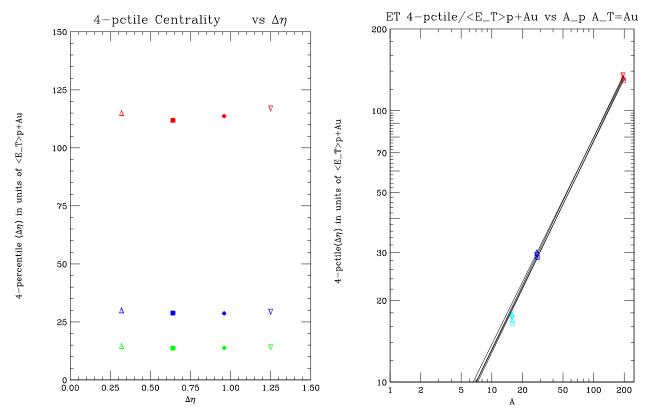


Figure 14: (left) 4%-ile of  $E_T$  distributions as a function  $\delta\eta$ , measured in units of  $\langle E_T(\delta\eta)\rangle_{p+A}$ : Au+Au (red), Si+Au (blue), O+Cu (green). (right)  $A_p$ -dependence of 4%-ile of  $E_T$  distributions as a function  $\delta\eta$ , measured in units of  $\langle E_T(\delta\eta)\rangle_{p+Au}$ . Lines are fits to  $A_p^{\alpha_p}$  from Si+Au to Au+Au (corrected to 14.6 A GeV/c). Cyan points are O+Au from WPNM.

- $\heartsuit$  4%-ile centrality cut is independent of  $\delta\eta$ , when measured in units of  $\langle E_T(\delta\eta)\rangle_{p+Au}$ .
- $\heartsuit$  The same is true for all centralities studied in this work: 7%-ile, 2%-ile, 1%-ile, 0.5%-ile.
- $\heartsuit$  With projectile dependence represented as  $A_p^{\alpha_p}$ : from O+Au to Si+Au,  $\alpha_p = 0.94 1.00$  depending on %-ile and  $\delta\eta$ ; from Si+Au to Au+Au,  $\alpha_p = 0.73 0.78$  depending on %-ile and  $\delta\eta$ .
- $\heartsuit$  Stachel at QM'90 showed  $A_T^{0.4}$  for S, Si, at both CERN and BNL energies. Any other data for  $A_p^{\alpha_p}$ ??

#### **CONCLUSIONS**

- Shapes of  $E_T$  distributions change with  $\delta \eta$  interval
- The shape  $\Gamma(p, b)$  and change of shape with  $\delta \eta$  is **Identical** for p+Au and p+Be in E802 for  $0.2 \le \delta \eta \le 1.25$ , around mid-rapidity.
- The shape (or fluctuation) of multiplicity distributions as parameterized by Normalized Factorial Moments or the NBD parameter  $1/k = K_2$  can be related to 2-particle correlations by an elegant theoretical framework; but we could find no such framework for the Gamma Distribution parameter  $p(\delta \eta)$  nor for  $E_T$  correlations.
- The Wounded Projectile Nucleon model works remarkably well to relate all the measured spectral shapes of Electromagnetic  $E_T$  distributions from p+Au, to O+Cu, to Si+Au to Au+Au at AGS energies for pseudorapidity intervals  $\delta \eta$  in the range  $0.2 \leq \delta \eta \leq 1.25$ .
- It is clear that  $E_T$  distributions in limited regions of  $\delta \eta$  provide an excellent characterization of the 'nuclear geometry' of RHI collisions, from which important information about the dynamics can be inferred.
- A new way of plotting  $E_T$  distributions was found. If the distribution in any aperture is plotted in units of the measured  $\langle E_T \rangle$  in the same aperture for p+Au collisions (or p-p, if available), the dynamics of the reaction, in terms of projectile (or total) participants, can be read directly from the plot.
- At AGS energies, the overall production of particles as observed by midrapidity  $E_T$  distributions may be interpreted as arising from incoherent nucleon-nucleus collisions, with the further implication that the stopping of the participant nucleons observed in central Au+Au collisions must be related to the identical shapes and evolution of the  $E_T$  distributions for p+Au and p+Be. In other words, the 'stopping' should be observable in p+A 'central' collisions.

### Distribution of Projectile Participants at a fixed $E_T$

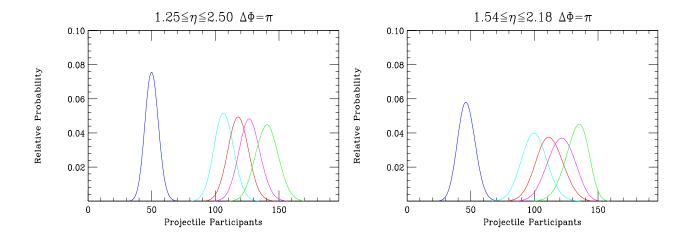
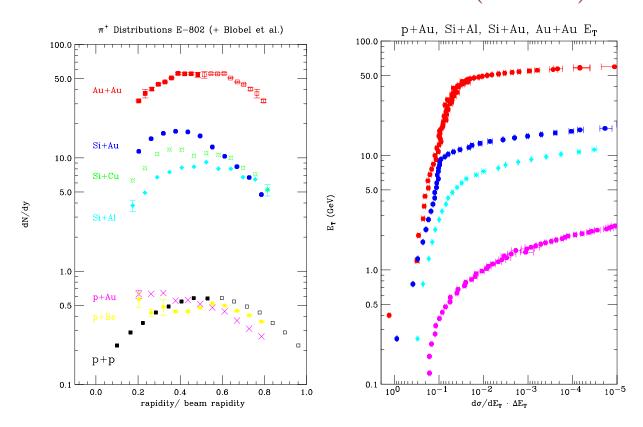


Figure 15: Distribution in Projectile Participants as a function of measured  $E_T$  ( $\Delta \phi = \pi$ ) for two  $\delta \eta$  intervals : (left)  $1.25 \leq \delta \eta \leq 2.50$ , (right)  $1.54 \leq \delta \eta \leq 2.18$ . The distributions are for five values of  $E_T$  corresponding to 31%-ile, 7%-ile, 4%-ile, 2%-ile, 1/2%-ile.

 $\heartsuit$  For a fixed  $E_T$  (measured with  $\leq 1\%$  resolution), the number of projectile participants varies by 7-9% (rms) (depending on the  $\delta\eta$ ) around the mean ( $\mu$ ) at the upper percentiles where 'centrality' is normally defined, increasing to a variation of  $\sigma = 10-15\%$  about the mean at the lower centrality values ( $\sim 50$  projectile participants), with continued proportionality  $\sigma/\mu \propto 1/\sqrt{\mu}$  at lower values.

 $\heartsuit$  It is hard to imagine that changes of any microscopic physical quantity as a function of measured  $E_T$  could be 'sharper' than the variation in Projectile Participants at the same  $E_T$  value. Then, of course, measurement resolution further smears the sharpness.

### Systematic measurements of dn/dy and $E_T$ in A+A collisions at the AGS (E802...)



On the left, the  $dn/dy|_{\pi^+}$  distributions for p+A inclusive and A+A central track the  $E_T$  distributions very well. It easy to see that Au+Au/p-p ~Au+Au/p+Au=100, much less than 197. Also the p-p or p+Be to p+Au ratios for  $\pi^+$  evidently don't follow the wounded nucleon model, since dn/dydoesn't vary at mid rapidity, but it does vary in the beam fragmentation region. The comparison Au+Au/Si+Al is intriguing since the Au+Au/Si+Al data for both dn/dy and  $E_T$  (right) are close to the WNM ratio of 7. However, from the  $E_T$  distributions, it is obvious that the exact value of this ratio depends on the "centrality" cut; since the "shapes" of the distributions vary The asymmetric Si+Au,Cu collisions show very interesting with A and  $\delta \eta$ . systematic behavior c.f. Si+Al, but may be a problem for a mid-rapidity calorimeter since the maximum in dn/dy moves out of the acceptance region. However, as the target dependence of the reaction is clearly emphasized by measurements in the target fragmentation region, while the projectile dependence is emphasized in the projectile fragmentation region, mid-rapidity measurements might represent a reasonable global average.

#### 5 Years of Calibrations

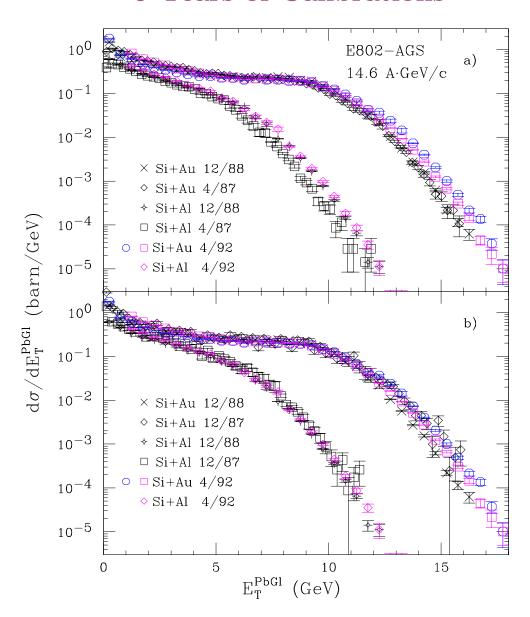


Fig. 11+1992 data

Figure 16:  $E_T$  distributions ( $\Delta \phi = \pi$ , 1.25  $\leq \delta \eta \leq$  2.50) for Si+Al and Si+Au from the April 1992 run (which includes Au+Au measurement), compared to measurements in previous runs [PRC 45, 2933 (1992), Fig. 11].

 $\heartsuit$  The 4/92  $E_T$  scale is 3% higher than 12/88, but in excellent agreement with 12/87.  $\circ$  and  $\square$  are 2 analyses of the same data.

## Yes, NA5 Mid-Rapidity $E_T$ Distributions change in shape with $\Delta\Phi$

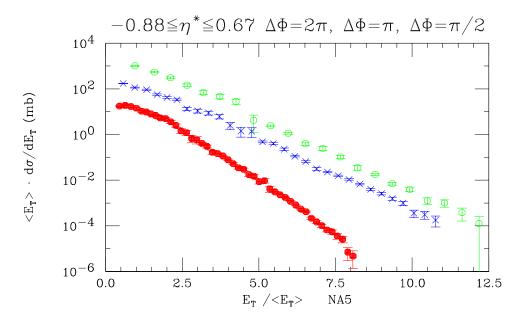


Figure 17: Original NA5  $E_T$  distribution (p-p at 300 GeV) for  $\Delta \phi = 2\pi$ ,  $\pi$ ,  $\pi/2$  scaled by  $\langle E_T \rangle$  in each aperture for  $-0.88 \le \eta^* \le 0.67$ . [PL **112B**, 173 (1982)].

### Distributions, Factorial Moments, Cumulants and the Two-Particle Rapidity Correlation

• In short-range rapidity (or pseudo-rapidity) correlation analyses, the normalized two-particle correlation function is usually taken as an exponential:

$$R_2(y_1, y_2) = \frac{\rho_2(y_1, y_2)}{\rho_1(y_1)\rho_1(y_2)} - 1 = R(0, 0) e^{-|y_1 - y_2|/\xi} \quad , \tag{1}$$

where  $\rho_1(y)$  and  $\rho_2(y_1, y_2)$  are the inclusive densities for a single particle (at rapidity y) or 2 particles (at rapidities  $y_1$  and  $y_2$ ), and  $\xi$  is the two-particle short-range rapidity correlation length.

• The two-particle normalized factorial moment (cumulant),  $F_2$  ( $K_2$ ), is nothing other than the integral of the two-particle correlation function on the interval:

$$K_2(\delta\eta) = F_2(\delta\eta) - 1 = \frac{\int^{\delta\eta} dy_1 dy_2 \, \rho_1(y_1) \rho_1(y_2) R_2(y_1, y_2)}{\int^{\delta\eta} dy_1 dy_2 \, \rho_1(y_1) \rho_1(y_2)} \quad . \tag{2}$$

- The integrand in the numerator, is the Mueller correlation function,  $C_2(y_1, y_2) = \rho_1(y_1)\rho_1(y_2)R_2(y_1, y_2) = \rho_2(y_1, y_2) \rho_1(y_1)\rho_1(y_2)$ , which is used in the discussion of conventional short-range correlations in multiparticle physics. The denominator of Eq. 2 is simply  $\langle n(\delta \eta) \rangle^2$ , the square of the mean multiplicity on the interval.
- For the case of a NBD,  $K_2(\delta \eta) = 1/k(\delta \eta)$ .
- If the inclusive single particle density,  $\rho_1(y) = dn/dy$ , is assumed constant on the interval, then the integral can be performed analytically (specifically on the interval  $0 \le y_1 \le \delta \eta$ ,  $0 \le y_2 \le \delta \eta$ ) to obtain the normalized factorial moment  $F_2(\delta \eta)$  or normalized factorial cumulant  $K_2(\delta \eta)$  in terms of the parameters of Eq. 1:

$$K_2(\delta \eta) = R(0,0) G(\delta \eta/\xi) \quad , \tag{3}$$

where the function G(x) is defined as:

$$G(x) = 2\frac{(x - 1 + e^{-x})}{x^2} . (4)$$

#### Demonstration that the two-pion correlation in rapidity is entirely due to Bose-Einstein correlation

